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13. ABSTRACT (Maximum 200 words) <p>Summary. This project focused on the interaction between geometric objects and combinatorial structures, especially <i>intersection graphs</i> and <i>containment orders</i>. Let Σ denote a family of sets. We call a graph G a Σ-<i>intersection graph</i> provided there is a mapping $f : V(G) \rightarrow \Sigma$ with the property that $uv \in E(G)$ exactly when $f(u) \cap f(v) \neq \emptyset$. Similarly, we call a partially ordered set P a Σ-<i>containment order</i> provided there is a mapping $f : P \rightarrow \Sigma$ so that $x \leq y$ exactly when $f(x) \subseteq f(y)$. A second theme in the research was the use of random methods and the development of novel models and applications of random graphs, including intermingling the intersection and random graph paradigms.</p> <p>The following is a list of publications resulting from this research:</p> <p>19960523 040</p>				
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Final Report

Intersection Graphs and Geometric/Combinatorial Optimization

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Summary. This project focused on the interaction between geometric objects and combinatorial structures, especially *intersection graphs* and *containment orders*. Let Σ denote a family of sets. We call a graph G a Σ -*intersection graph* provided there is a mapping $f : V(G) \rightarrow \Sigma$ with the property that $uv \in E(G)$ exactly when $f(u) \cap f(v) \neq \emptyset$. Similarly, we call a partially ordered set P a Σ -*containment order* provided there is a mapping $f : P \rightarrow \Sigma$ so that $x \leq y$ exactly when $f(x) \subseteq f(y)$. A second theme in the research was the use of random methods and the development of novel models and applications of random graphs, including intermingling the intersection and random graph paradigms.

The following is a list of publications resulting from this research:

- On the structure of hereditary classes of graphs, *Journal of Graph Theory* **10** (1986) 545–551.
- Irredundancy in multiple interval representations, *Discrete Mathematics* **63** (1987) 101–108.
- (with J. Gimbel, et. al.) Hamiltonian closure in random graphs, *Annals of Discrete Mathematics* **33** (1987) 59–67.
- (with J. Wierman) Infinite AB percolation clusters exist, *Journal of Physics A* **20** (1987) 1305–1307.
- The maximum interval number of graphs with given genus, *Journal of Graph Theory* **11** (1987) 441–446.
- Almost sure fault tolerance in random graphs, *SIAM Journal on Computing* **16** (1987) 1124–1134.
- (with J. Wierman) On circle containment orders, *Order* **4** (1988) 315–318.
- (with N. Alon) Degrees of freedom versus dimension for containment orders, *Order* **5** (1988) 11–16.
- Random interval graphs, *Combinatorica* **8** (1988) 357–371.
- On the interval number of a chordal graph, *Journal of Graph Theory* **12** (1988) 311–316.

- (with M. Golumbic) Containment graphs, posets and related classes of graphs, *Combinatorial Mathematics: Proceedings of the Third International Conference*, Annals of the New York Academy of Sciences **555** (1989) 192–204.
- (with D. West) Interval representations of cliques and subset intersection graphs, *Combinatorial Mathematics: Proceedings of the Third International Conference*, Annals of the New York Academy of Sciences **555** (1989) 363–367.
- (with J. Wierman) Optimal and near-optimal broadcast in random graphs, *Discrete Applied Mathematics* **25** (1989) 289–297.
- On the expected capacity of binomial and random concentrators, *SIAM Journal on Computing* **19** (1990) 156–163.
- (with N. Alon and M. Katchalski) Not all graphs are segment T-graphs, *European Journal of Combinatorics* **11** (1990) 7–13.
- On the interval number of random graphs, *Discrete Mathematics* **82** (1990) 105–109.
- An evolution of interval graphs, *Discrete Mathematics* **82** (1990) 287–302.
- (with A. Trenk) On generalized perfect graphs: α -perfection and acyclic χ -perfection, *Congressus Numerantium* **79** (1990) 163–172.
- (with J. Justicz and P. Winkler) Random intervals, *American Mathematical Monthly* **97** (December 1990) 881–889.
- (with A. Fraenkel) A deletion game on graphs, *Discrete Applied Mathematics* **30** (1991) 155–162.
- (with A. Dean and J. Hutchinson) On the thickness and arboricity of a graph, *Journal of Combinatorial Theory B* **52** (1991) 147–151.
- A note on planar graphs and circle orders, *SIAM Journal on Discrete Mathematics* **4** (1991) 448–451.
- (with F.R. McMorris) Connectivity threshold for random chordal graphs, *Graphs and Combinatorics* **7** (1991) 177–181.

- (with M. Jacobson and F.R. McMorris) General results on tolerance intersection graphs, *Journal of Graph Theory* **15** (1991) 573–577.
- Generalized chromatic number of random graphs, *SIAM Journal on Discrete Mathematics* **5** (1992) 74–80.
- (with N. Alon) Generalized sum graphs, *Graphs and Combinatorics* **8** (1992) 23–29.
- (with G. Brightwell) On the fractional dimension of partial orders, *Order* **9** (1992) 139–158.
- The many faces of circle orders, *Order* **9** (1992) 343–348.
- (with T. McKee) On the chordality of a graph, *Journal of Graph Theory* **17** (1993) 221–232.
- (with A. Trenk) On generalized perfect graphs: bounded degree and bounded edge perfection, *Discrete Applied Mathematics* **44** (1993) 233–245.
- (with G. Brightwell) Representations of planar graphs, *SIAM J. Discr. Math.* **6** (1993) 214–229.
- A note on graphs and sphere orders, *Journal of Graph Theory* **17** (1993) 283–290.
- (with A.S. Fraenkel and D. Ullman) Undirected edge geography, *Theoretical Computer Science* **112** (1993) 371–382.
- (with M. Sapir) Irrepresentability of short semilattices by Euclidean subspaces, *Algebra Universalis* **31** (1994) 599–607.
- (with J. Zito) On the size of hereditary properties of graphs, *Journal of Combinatorial Theory (B)* **61** (1994) 16–39.
- (with M. Ramana and D. Ullman) Fractional isomorphism of graphs, *Discrete Mathematics* **132** (1994) 247–265.
- (with H.S. Wilf) The rectilinear crossing number of a complete graph and Sylvester’s “four point problem” of geometric probability, *American Mathematical Monthly* **101** (December 1994) 939–943.

- (with A. Trenk and D. Ullman) On point-halfspace graphs, *Journal of Graph Theory*, to appear.
- (with G. Brightwell) The dual of a circle order is not necessarily a circle order, *Ars Combinatoria*, to appear.

The fruits of this research were presented at a variety of professional meetings and university research seminars and colloquia. Here is a list of talks (invited and contributed) given by the Principal Investigator on research conducted in connection with this grant during the course of the grant:

Invited Addresses

- University of Pennsylvania, Mathematics, 1986.
- Southern Methodist University, Computer Sciences, 1986.
- University of Maryland, Baltimore County, Mathematics, 1987.
- The Weizmann Institute, Combinatorics Conference, 1987.
- The Technion, Mathematics, Computer Science, 1987.
- SIAM National Meeting, Denver, Colorado, November, 1987.
- George Washington University, Mathematics, February, 1988, September, 1990.
- Interface '88 Conference, Reston, Virginia, April, 1988.
- ORSA-TIMS National Conference, Washington, D.C., April, 1988.
- University of Louisville, December, 1988.
- Wright State University, April, 1990, (March, 1993).
- Fifth SIAM Conference on Discrete Mathematics, Atlanta, June, 1990.
- Bellcore, October, 1990.
- London School of Economics, December, 1990.

- University of Illinois—Chicago, April, 1991.
- The Sixth Clemson Miniconference on Discrete Mathematics, October, 1991.
- Workshop on Planar Graphs, DIMACS (Rutgers), November, 1991.
- Emory University, December, 1991.
- Dartmouth College, October, 1992.
- University of Pennsylvania, Mathematics, November, 1992.
- Loyola College, Baltimore, Maryland, March, 1993.
- MIGHTY Conference, Miami University, Ohio, March, 1993.
- AMS Regional Conference, Washington, D.C., April, 1993.
- University of Delaware, April, 1993.
- London School of Economics, July, 1993.
- Cambridge University, July, 1993.
- Adam Mickiewicz University (Poznań, Poland), January, 1994.
- Supercomputing Research Center, August, 1994.
- University of Toronto, October, 1994.

Contributed Addresses

- The Southeastern Conference on Combinatorics, Graph Theory and Computing (Florida Atlantic University), 1985, 1986, 1989.
- The Two Hundred Fiftieth Anniversary Conference on Graph Theory (Indiana University—Purdue University at Fort Wayne), 1986.
- ARIDAM II, Rutgers Center for Operations Research, 1987.
- The Eleventh British Combinatorial Conference (Goldsmiths College, University of London), 1987.

- Sixth International Conference on the Theory and Applications of Graphs, Western Michigan University, June, 1988.
- Fourth SIAM Conference on Discrete Mathematics, San Francisco, June, 1988.
- Fourth International Colloquium on the Theory of Graphs, and Combinatorics, Marseille, France, July, 1990.
- Thirteenth British Combinatorial Conference, United Kingdom, July, 1991.
- Seventh International Conference on the Theory and Applications of Graphs, Western Michigan University, June, 1992.
- Sixth SIAM Conference on Discrete Mathematics, Vancouver, June, 1992.
- Fourteenth British Combinatorial Conference, University of Keele, July, 1993.